Computer-generated Control Lyapunov Function via offline linear programming

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Outline

1. Computer generated Control Lyapunov Function
   - Motivation and main idea
   - Illustrative example
   - Experimental validation on quadcopter stabilization
2. Conclusion and future direction

Outline

Control Lyapunov Function

Consider a control affine system:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u}$$

CLF $V(\boldsymbol{x})$

$\Rightarrow$ find $\boldsymbol{u} \in \{\dot{V}(\boldsymbol{x}) \leq -\beta V(\boldsymbol{x})\}$

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via "universal" formulas

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Generalized framework: space triangulation [Steentjes et al., 2020, Lavaei and Bridgeman, 2023],
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Computationally costly to artificially generate a CLF

Model Predictive Control

Consider a standard MPC design:

$$
\begin{aligned}
&\arg\min_{\boldsymbol{u}(\cdot)} \sum_{k=0}^{N_p} \left(\|\boldsymbol{x}(k)\|^2_{\boldsymbol{Q}} + \|\boldsymbol{u}(k)\|^2_{\boldsymbol{R}}\right) + \|\boldsymbol{x}(N_p)\|^2_{\boldsymbol{P}} \\
&\text{s.t} \begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{x}(k) + \tau\left[f(\boldsymbol{x}(k)) + g(\boldsymbol{x}(k))\boldsymbol{u}(k)\right], \\ \tau \text{ is the sampling time, } \boldsymbol{u}(k) \in \mathcal{U}, \\ \boldsymbol{x}(k) \in \mathcal{X}, \boldsymbol{x}(N_p) \in \mathcal{X}_f, \end{cases}
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MPC: an alternative method to sidestep CLF design

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CLF-based control	versus	**MPC**
Complicated offline synthesis		Systematic offline design
Straightforward control implementation		Computationally demanding online implementation

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Complicated offline synthesis

Straightforward control implementation

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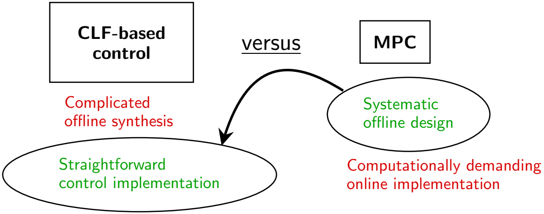
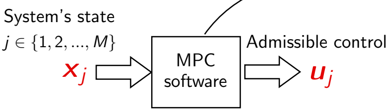
Main idea



Main idea: Use the admissible control from the MPC law to parameterize a CLF.

CLF offline synthesis based on Linear Program

- **Step 1: Generate and collect admissible points in state-input space**



$$\arg\min_{\boldsymbol{u}(\cdot)} \sum_{k=0}^{N_p} \left(\|\boldsymbol{x}(k)\|_{\boldsymbol{Q}}^2 + \|\boldsymbol{u}(k)\|_{\boldsymbol{R}}^2 \right) + \|\boldsymbol{x}(N_p)\|_{\boldsymbol{P}}^2$$

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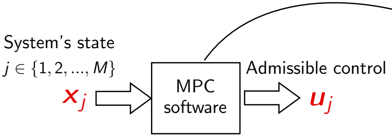

$$\underset{\boldsymbol{u}(\cdot)}{\arg\min} \sum_{k=0}^{N_p} \left(\|\boldsymbol{x}(k)\|_{\boldsymbol{Q}}^2 + \|\boldsymbol{u}(k)\|_{\boldsymbol{R}}^2 \right) + \|\boldsymbol{x}(N_p)\|_{\boldsymbol{P}}^2$$
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- **Step 2: Fix a collection of basis candidate CLFs $V_i(\boldsymbol{x})$ and define:**

$$V(\boldsymbol{x}) = \sum_{i=1}^{N} \alpha_i V_i(\boldsymbol{x})$$

where $\alpha_i \geq 0, \sum_{i=1}^{N} \alpha_i = 1$, are the scalar coefficients parameterizing the CLF to find later.

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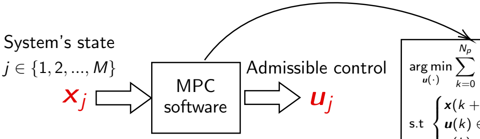

$$\arg\min_{u(\cdot)} \sum_{k=0}^{N_p} \left(\|x(k)\|_Q^2 + \|u(k)\|_R^2 \right) + \|x(N_p)\|_P^2$$

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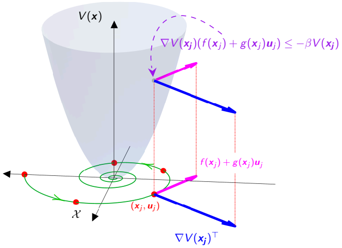
- **Step 3: Find the coefficients α_i via the following linear program, for some $\beta > 0$:**

$$\min_{\alpha_i} \sum_{i=1}^{N} \ell_i \alpha_i$$

$$\sum_{i=1}^{N} \alpha_i \left[\nabla V_i(x_j) \left[f(x_j) + g(x_j) u_j \right] + \beta V_i(x_j) \right] \leq 0,$$

$$\sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0, j \in \{1, ..., M\}.$$

where $\ell_i > 0$, are user-defined scalar weighting, prioritizing the dominance of $V_i(x)$.



Illustrative example

Consider the double integrator system:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu$$

$\boldsymbol{x} = [x_1, x_2]^\top, |u| \leq 0.8154,$

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = [0 \quad 1]^\top.$

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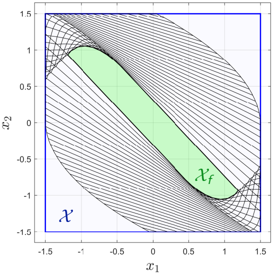
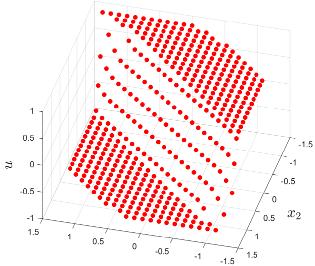
Step 1: collect admissible pairs from the MPC software

Discretized MPC design:

$$\arg\min_{u(\cdot)} \sum_{k=0}^{N_p} \left(\|\boldsymbol{x}(k)\|^2_{\boldsymbol{Q}} + \|\boldsymbol{u}(k)\|^2_{\boldsymbol{R}} \right) + \|\boldsymbol{x}(N_p)\|^2_{\boldsymbol{P}}$$

$$\text{s.t} \begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{x}(k) + \tau \left[A\boldsymbol{x}(k) + Bu(k)\right], \\ \boldsymbol{u}(k) \in \mathcal{U}, \boldsymbol{x}(k) \in \mathcal{X}, \boldsymbol{x}(N_p) \in \mathcal{X}_f, \end{cases}$$





with $\boldsymbol{Q} = \text{diag}(50, 5), \boldsymbol{R} = 5, \boldsymbol{P} = \begin{bmatrix} 479.6118 & 181.0469 \\ 181.0469 & 155.5598 \end{bmatrix}$, $\mathcal{X}_f$ is the maximal positive invariant set with the local control $u = -[2.7617 \ 2.6491]\boldsymbol{x}$.

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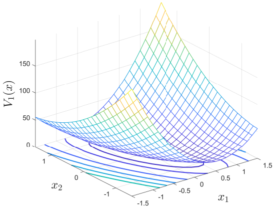
The first member:

$$V_1(\boldsymbol{x}) = \boldsymbol{x}^\top \boldsymbol{P}_{lqr}\boldsymbol{x},$$

a quadratic CLF (for optimality).



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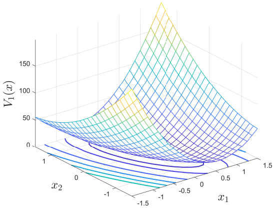
The other members: $2p$-norm CLF (for expansion of the domain of attraction) [Blanchini and Miani, 1999]:

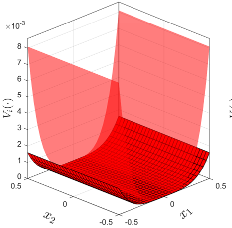
$$V_i(\boldsymbol{x}) = \|\boldsymbol{G}\boldsymbol{x}\|_{2p}^{2p},$$

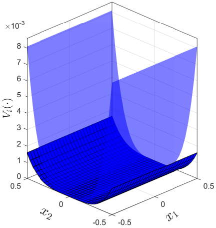
$\boldsymbol{G}$ is a full row rank.

$$V_{2\le i\le 100}(\boldsymbol{x}) \in \Big\{ \|\eta \boldsymbol{F}\boldsymbol{x}\|_4^4, \eta \in \text{linspace}(33, 0.4, 2.0)$$

$$\boldsymbol{F} \in \{[1 \quad 0], [0 \quad 1], [1 \quad 1]\} \Big\}.$$







Illustrative example

Step 3: Find the CLF

$V(\boldsymbol{x}) = \sum_{i=1}^{N} \alpha_i V_i(\boldsymbol{x})$.

$$\alpha_i^* = \arg\min_{\alpha_i} \sum_{i=1}^{N} \ell_i \alpha_i$$

Parameters	Value
ℓ_1	0.05
$\ell_{2\le i\le 100}$	0.1
β	0.1275
N^o of basis functions N	100
N^o of sampled pairs M	625

$$\sum_{i=1}^{N} \alpha_i \left[\nabla V_i(\boldsymbol{x}_j)\left[f(\boldsymbol{x}_j) + g(\boldsymbol{x}_j)\boldsymbol{u}_j\right] + \beta V_i(\boldsymbol{x}_j)\right] \le 0; \sum_{i=1}^{N} \alpha_i = 1; \alpha_i \ge 0, j \in \{1, ..., M\}.$$

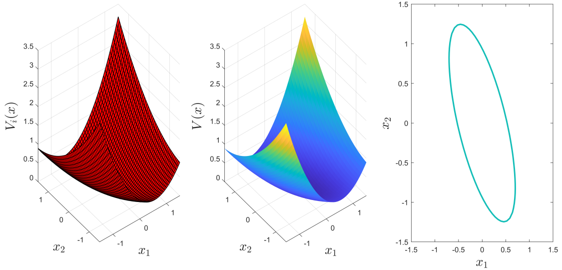


Figure 1: (left) Member functions (middle) the generated CLF and (right) the domain of attraction

$$V(\boldsymbol{x}) = 0.016\|\boldsymbol{x}\|^2_{\boldsymbol{P}_{lqr}} + 0.2293(2[1\ 0]\boldsymbol{x})^4 + 0.5406(2[0\ 1]\boldsymbol{x})^4 + 0.2141(2[1\ 1]\boldsymbol{x})^4.$$

solver: `linprog`, MATLAB 2021b, solver time: 0.0315 (sec).

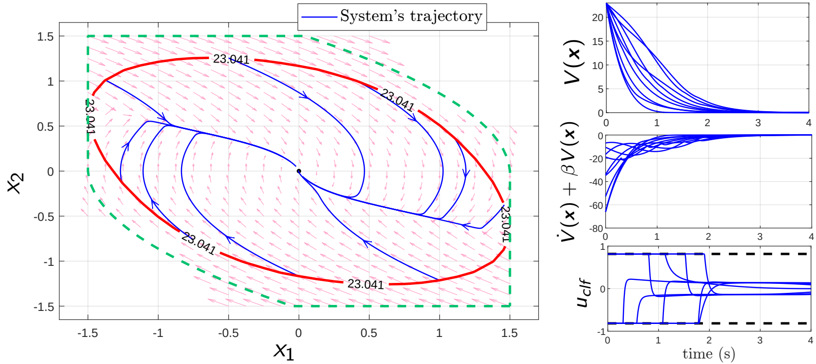
Simulation

Controller implementation:

$$u_{clf}(\boldsymbol{x}) = \arg\min_{u} \|\boldsymbol{u} - \boldsymbol{u}_d(\boldsymbol{x})\|^2$$

$$\text{s.t} \begin{cases} \boldsymbol{u} \in \mathcal{U}, \\ \nabla V(\boldsymbol{x})(f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u}) \leq -\beta V(\boldsymbol{x}). \end{cases}$$

where $\boldsymbol{u}_d(\boldsymbol{x})$ is a user-defined desired control to avoid slow performance due to possibly small β.



Validation with quadcopter outer-loop control

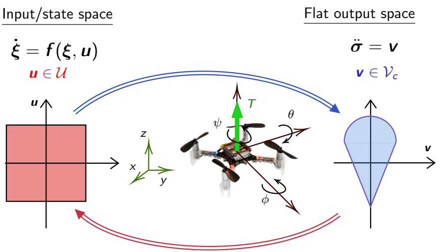


Figure 2: Constraint deformation due to the flatness-based feedback linearization.

Consider the quadcopter's outer-loop control:

$$\dot{\boldsymbol{\xi}} = A\boldsymbol{\xi} + B(\boldsymbol{R}_\psi \boldsymbol{f}(\boldsymbol{u}) - g\boldsymbol{e}_3)$$

$$\boldsymbol{\sigma} = [x, y, z]^\top \in \mathbb{R}^3,\ \boldsymbol{\xi} = [\boldsymbol{\sigma}^\top, \dot{\boldsymbol{\sigma}}^\top]^\top$$

$$\boldsymbol{u} = [T, \phi, \theta]^\top \in \mathcal{U} = \{0 \le T \le T_{max},\ |\phi|, |\theta| \le \epsilon_{max}\}$$

Feedback linearization[1] $\quad \boldsymbol{u} = \boldsymbol{f}^{-1}(\boldsymbol{R}_\psi^{-1}(\boldsymbol{v} + g\boldsymbol{e}_3))$

$$\ddot{\boldsymbol{\sigma}} = \boldsymbol{v} = [v_1, v_2, v_3]^\top$$

$$\boldsymbol{v} \in \mathcal{V}_c = \left\{\|\boldsymbol{v} + g\boldsymbol{e}_3\| \le T_{max},\ \sqrt{v_1^2 + v_2^2} \le \tan \epsilon_{max}(v_3 + g)\right\}.$$

[1] H.T. Do and I. Prodan, "Indoor experimental validation of MPC-based trajectory tracking for a quadcopter via a flat mapping approach," 2023 European Control Conference, Bucharest, Romania.

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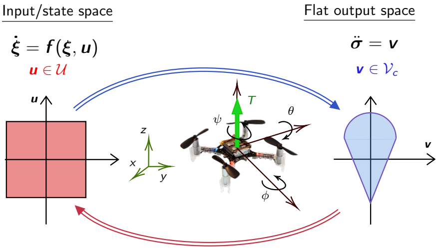

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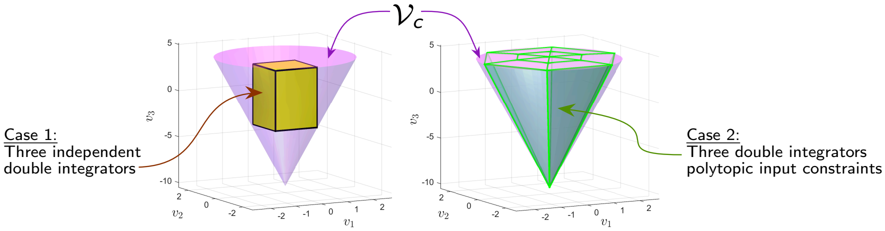

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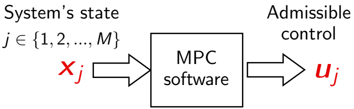


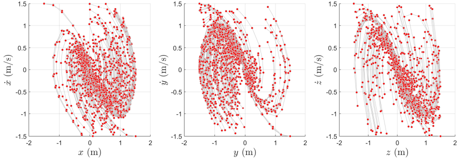


Figure 3: Sampled admissible pairs from MPC for Case 2.

Step 2 & 3: fix the basis functions and solve the LP for the CLFs

Case 1: The system is decoupled into three separated double integrators:

$$\ddot{x} = v_1, \ddot{y} = v_2, \ddot{z} = v_3,$$

with $|v_i| \leq \bar{v}_i, i \in \{1, 2, 3\}$. The CLFs:

- For $q \in \{x, y\}$ axis:
  $$V^q(\boldsymbol{x}) = 0.016\|\boldsymbol{x}\|^2_{\boldsymbol{P}_{lqr}} + 0.2293(2[1\,0]\boldsymbol{x})^4 + 0.5406(2[0\,1]\boldsymbol{x})^4 + 0.2141(2[1\,1]\boldsymbol{x})^4.$$
- For the z axis:
  $$V^z(\boldsymbol{x}) = 0.0119\|\boldsymbol{x}\|^2_{\boldsymbol{P}_{lqr}} + 0.3862(2[1\,0]\boldsymbol{x})^4 + 0.6019(2[1\,1]\boldsymbol{x})^4.$$

Case 2: The system as three concatenated double integrators:

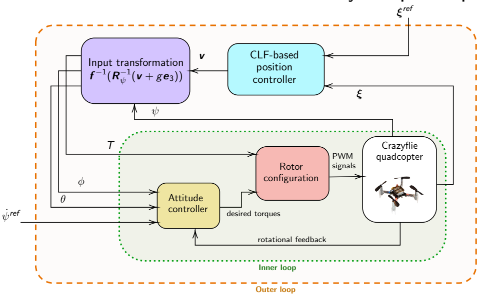
$$\dot{\boldsymbol{\xi}} = A\boldsymbol{\xi} + B\boldsymbol{v},$$

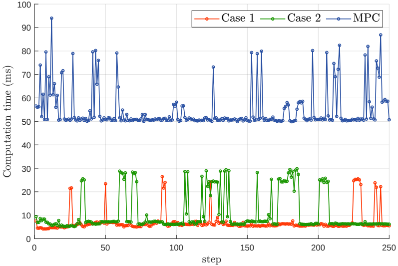
with $H\boldsymbol{v} \leq h$.

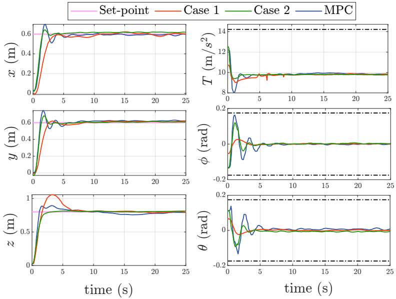
$$V(\boldsymbol{\xi}) = 0.0244\|\boldsymbol{\xi}\|^2_{\boldsymbol{P}_{lqr}} + 0.4508([0\,1\,0\,0\,0\,0]\boldsymbol{\xi})^4 + 0.0674([1\,1\,0\,0\,0\,0]\boldsymbol{\xi})^4 + 0.1631([0\,0\,1\,0\,0\,0]\boldsymbol{\xi})^4 + 0.1841([0\,0\,1\,1\,0\,0]\boldsymbol{\xi})^4 + 0.1103([0\,0\,0\,0\,0\,1]\boldsymbol{\xi})^4.$$

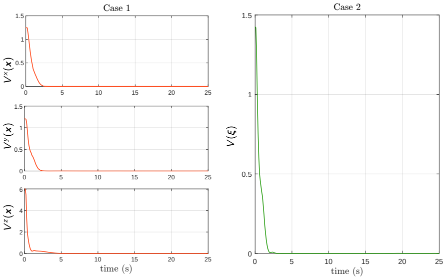
Set-point tracking, comparison with MPC

Hierarchical control structure of Crazyflie quadcopter:







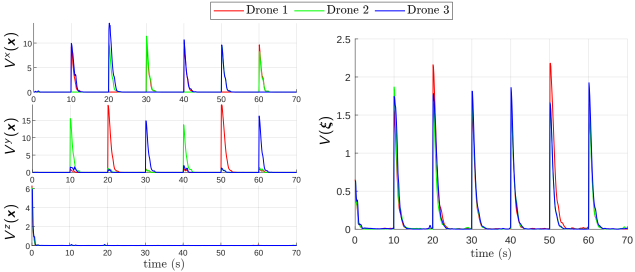


Multiple drones control

The low complexity of the CLF-based law allows controlling multiple drones in a centralized manner.

Table 1: Experiment results

	Controller	RMS tracking error (cm)			Avg. CT
Sce. 1	Case 1	14.11			7.02 ms
	Case 2	11.39			10.74 ms
	MPC	11.48			54.70 ms
Sce. 2		Drone 1	Drone 2	Drone 3	
	Case 1	24.09	22.20	21.33	24.09 ms
	Case 2	19.14	18.57	18.71	20.51 ms
	MPC	×	×	×	×



Experiment video: `https://youtu.be/PP5fZnIyH54`

Outline

Conclusion

Conclusion:

- A linear program-based technique was proposed to synthesize CLF with admissible state-input pairs collected from an MPC software.
- The result shows that the technique has low computational footprint during the offline synthesis, allowing the benefits of CLF-based online implementation.

Future work:

- Investigate how to efficiently choose the sampled data from MPC, providing theoretical guarantees over a finite set of constraints and allowing scalability.
- Include robustness analysis in the CLF synthesis.

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